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Explicit error bounds for multivariate analytic combinatorics

Joint work with Stephen Melczer (smelczer@uwaterloo.ca)

AMS Spring Sectional Meeting 2026, North Dakota State University, Fargo, ND

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$$F(\mathbf{z}) = \sum_{i_1, i_2, \dots, i_d \geq 0} f_{i_1, i_2, \dots, i_d} z_1^{i_1} z_2^{i_2} \cdots z_d^{i_d} = \sum_{\mathbf{i} \in \mathbb{N}^d} f_{\mathbf{i}} z^{\mathbf{i}}$$

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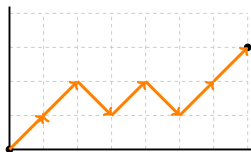
Can one compute asymptotics for the *diagonal coefficient sequence* $(f_{n\mathbf{r}})_{n=0}^{\infty}$?

Applications

Diagonals of rational functions appear in the study of...

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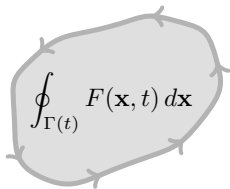
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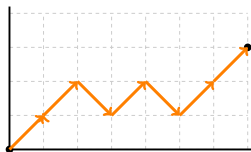
Lattice paths

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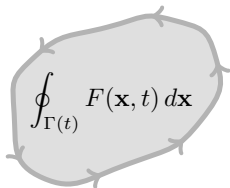
Period integrals



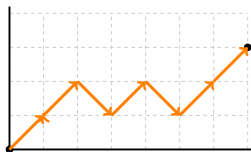
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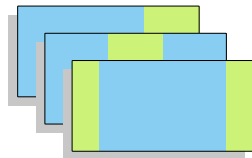
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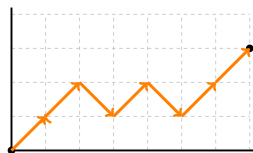
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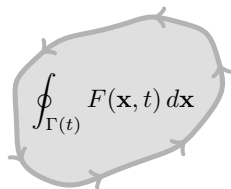
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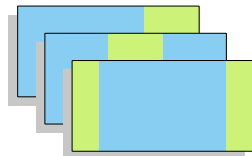
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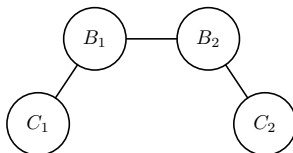
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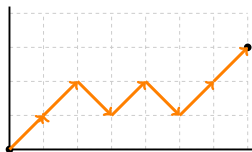
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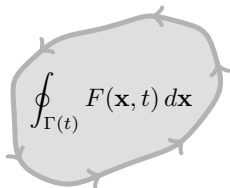
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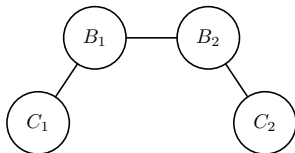


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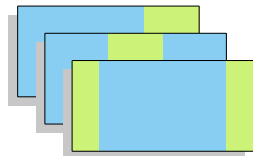


Period integrals

...AND MUCH MORE!



Graphs and Networks



Irrational tilings

Another Problem: Positivity

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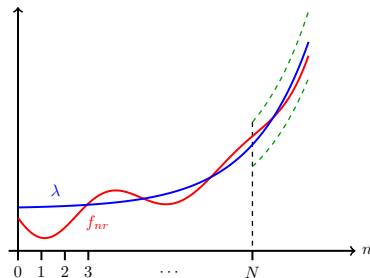
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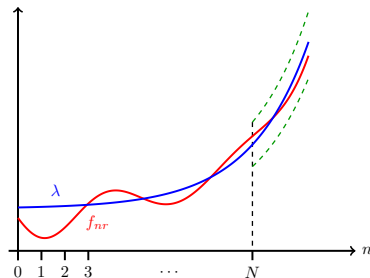


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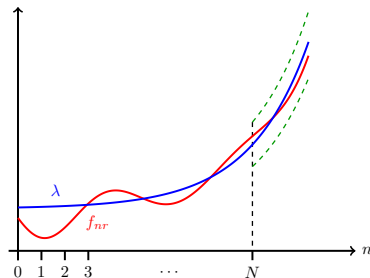


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This motivates the need for explicit error bounds!

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Software:[illegible]

See `bound_coefficients` in `Ore_Algebra` package, or [DMM 2023].

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- How do we compute $\lambda(n)$ and $N \in \mathbb{N}$ such that

$$|f_{n\mathbf{r}} - \lambda(n)| \leq \frac{1}{2} |\lambda(n)|$$

for all $n \geq N$?

Answer: Take inspiration from the univariate case!

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- **Step 4:** Find *minimal critical* points.

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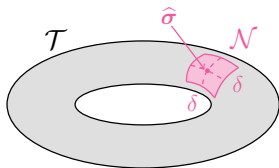
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- *finitely minimal* if $\{H(z) = 0\} \cap T(\sigma)$ is finite.

Assuming strictly minimal critical point σ with “nice” geometry:

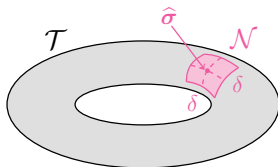
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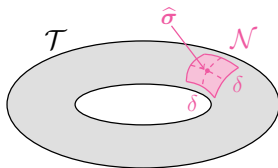
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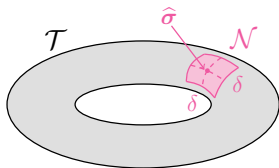
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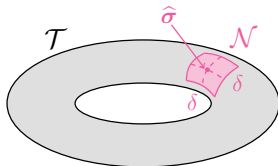


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$$\chi = \frac{\sigma^{-nr}}{(2\pi)^{d-1}} \int_{[-\delta, \delta]^{d-1}} A(\theta) e^{-n\phi(\theta)} d\theta$$

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Proposition 1

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A similar result holds in the finitely minimal case.

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Proposition 2

Let σ, χ be as above with σ nondegenerate. Let $\alpha \in \mathbb{Q} \cap (1/3, 1/2)$ and define

$$\lambda(n) := \frac{\sigma^{-nr}}{n^{(d-1)/2}} \frac{(2\pi)^{(d-1)/2} A(\mathbf{0})}{\sqrt{\det \mathcal{H}}}.$$

Then one can construct $N \in \mathbb{N}$ and $C > 0$ such that

$$|\chi - \lambda| \leq C|\lambda|n^{1-3\alpha}.$$

for all $n \geq N$.

Theorem

Let σ be a *strictly minimal nondegenerate smooth critical point*, and let f_{nr} , λ , α be as above. Then $\forall \epsilon \in (0, 1)$ there exists a computable $N \in \mathbb{N}$ such that

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Again, one can extend this argument to the finitely minimal case without much difficulty.

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Gillis-Reznick-Zeilberger

$$F(z_1, \dots, z_d) = \frac{1}{1 - \sum_i z_i + d! \prod_i z_i}, \quad d \geq 4.$$

With $d = 4$ we get $N > 10^6$.

We wrote a Sage package for computing diagonals...

```
[5]: from sage_periods import *
var('x y z w')
F = 1/(1-x-y-z-w + 24*x*y*z*w)

%time L = compute_diagonal_annihilator(F)
L

CPU times: user 896 ms, sys: 3.09 ms, total: 899 ms
Wall time: 897 ms

[5]: (t^8 + 5/36*t^7 + 31/5184*t^6 - 23/31104*t^5 + 55/2985984*t^4 - 19/107495424*t^3 + 1/1719926784*t^2)*Dt^3 + (6*t^7 + 13/24*t^6 + 1/192*t^5 - 37/20736*t^4 + 13/248832*t^3 - 13/23887872*t^2 + 1/573308928*t)*Dt^2 + (7*t^6 + 5/18*t^5 - 23/1728*t^4 - 1/6912*t^3 + 11/995328*t^2 - 7/35831808*t + 1/1719926784)*Dt + t^5 - 1/72*t^4 - 1/864*t^3 + 1/41472*t^2 - 1/995328*t

[11]: var('z')
F2 = 1/(1-x-y)
%time L2 = compute_diagonal_annihilator(F2,r=[2,3],vari=[x,y],t=z, Dt = "D_z")
show(L2)

CPU times: user 350 ms, sys: 3.12 ms, total: 353 ms
Wall time: 353 ms

$$\left(z^4 - \frac{108}{3125}z^3\right)D_z^4 + \left(8z^3 - \frac{486}{3125}z^2\right)D_z^3 + \left(\frac{72}{5}z^2 - \frac{348}{3125}z\right)D_z^2 + \left(\frac{24}{5}z - \frac{12}{3125}\right)D_z + \frac{24}{625}$$


[14]: diag = fast_diagonal_series(F2,z,10,[2,3])
diag

[14]: 1 + 10*z + 210*z^2 + 5005*z^3 + 125970*z^4 + 3268760*z^5 + 86493225*z^6 + 2319959400*z^7 + 62852101650*z^8 + 1715884494940*z^9 + 47129212243960*z^10 + 0(z^11)

[15]: L2(diag)

[15]: 0(z^10)
```

https://github.com/ACSVMath/sage_periods

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Thank you! :)